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Errors in Transitions Between Algebraic and Other Representations: A Qualitative Study of 7th and 8th Grade Students*

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Abstract

Mathematical understanding is closely related to how concepts are represented and how learners coordinate different forms of representation. Within this context, multiple representations constitute a central component of mathematics learning by enabling students to construct and communicate meaning. This study aims to examine 7th and 8th grade students' ability to transition between multiple representations (e.g., graphical, tabular, and verbal forms), focusing specifically on the errors made in transitions from algebraic representations to other forms and vice versa. This study was designed as a qualitative case study. The study group consisted of middle school students, and data were collected through open-ended tasks requiring students to translate between different representations. The analysis was guided by a multiple representation framework based on Lesh (1979) and Janvier (1987), and the data were analyzed using content analysis. The findings indicate that students experienced the greatest difficulty in transitions from graphical to algebraic representations. Many students failed to interpret key graphical features such as slope and intercept and translate them into algebraic expressions. While transitions between tabular and algebraic representations showed similar error patterns across grade levels, differences emerged in algebraic-graphical and algebraic-verbal transitions. The most common error was performing numerical calculations without constructing algebraic expressions. In many cases, students focused on computing specific values rather than generalizing symbolically. Overall, the results highlight significant difficulties in representational transitions and emphasize the need for instructional practices that explicitly support representational fluency and conceptual understanding.

Keywords: Multiple representations, representation translation, algebraic representation, errors, middle school

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Introduction

Mathematics plays an important role in many areas of daily life and supports individuals' reasoning and problem-solving skills; however, its abstract nature often leads students to perceive it as difficult and incomprehensible. This challenge arises not only from the complexity of mathematical concepts but also from how these concepts are presented. In this regard, the type and use of mathematical representations have a critical influence on learning (Goldin & Shteingold, 2001).

This study is grounded in a multiple representation theoretical perspective, which emphasizes that mathematical understanding develops through the coordination, transformation, and integration of different representational forms. Deep mathematical understanding requires learners to view concepts from multiple perspectives and express them through different forms. According to NCTM (2000), using multiple representations enhances learning and promotes mathematical thinking. Representations serve as tools that help individuals concretize and communicate abstract mathematical ideas (Delice & Sevimli, 2010). Goldin and Shteingold (2001) define multiple representations as expressing a mathematical idea in different ways. Recent international research highlights that students' representational fluency—their ability to move flexibly between representations—is a key determinant of conceptual understanding. However, students often struggle with coordinating multiple representations and maintaining consistency across them (McCulloch et al., 2022; Adom, 2020; Gülkilik et al., 2023).

Although representations are used across all mathematics domains, they are particularly central in algebra instruction (Confrey & Smith, 1991; Kaput, 1989; MEB, 2018). Research consistently shows that representational diversity enhances students' understanding of algebraic concepts. Stylianou (2010) found that interpreting graphs directly influences comprehension of algebraic relationships, while Blanton and Kaput (2005) emphasized the importance of transitioning between representations in the development of algebraic reasoning. In addition, recent studies emphasize that students tend to focus on surface-level features of representations rather than underlying mathematical relationships, which leads to persistent misconceptions and errors, especially in transitions between graphical and algebraic forms (McCulloch et al., 2022; Adom, 2020).

Representations are not merely tools for displaying information; they are integral to learning itself (NCTM, 2000). Using multiple representations deepens conceptual understanding (Even, 1998; Hiebert & Carpenter, 1992) and improves students' ability to transition flexibly between representational forms (Bosse et al., 2011). Furthermore, technology-enhanced learning environments provide dynamic opportunities for connecting representations; however, research shows that such environments do not automatically eliminate students' difficulties in representational transitions (Gülkilik et al., 2023).

In Türkiye, the concept of representation was formally included in the mathematics curriculum in 2013, and the 2018 program emphasized the importance of formulas, models, graphs, and tables. However, textbook analyses show that algebraic representations dominate, while graphs and real-life contexts receive limited attention (İncikabı & Biber, 2017). Studies in Türkiye indicate that multiple representations improve achievement and conceptual understanding (Yapıcı, 2013; Kaya, 2015; Çetin, 2024; Baran, 2024). However, students continue to experience significant difficulties, particularly with graphical representations (Yağız & Broutin, 2022). Despite the growing body of research, there remains a need for studies that systematically examine the nature, sources, and types of errors occurring during two-way transitions between representations, particularly at the middle school level. The 7th and 8th grades were selected because these years represent a critical stage in algebra learning, where students transition from arithmetic thinking to more abstract algebraic reasoning. This period is particularly sensitive for investigating representational fluency, as students begin to encounter multiple representation forms in both symbolic and graphical contexts.

Theoretical Framework

This study is based on the theory of multiple representations in mathematics education, which posits that mathematical knowledge is constructed through the interaction, coordination, and transformation of different representational forms.

Representation in Mathematics Learning

Representations are central to mathematical thinking and learning, as they enable learners to externalize, interpret, and communicate abstract ideas (NCTM, 2000). Goldin and Shteingold (2001) conceptualize representations as systems that encode mathematical meaning, while Lesh (1979) emphasizes their role in linking real-world situations with formal mathematical structures.

From a cognitive perspective, representations function as mediational tools that support meaning-making processes and reduce cognitive load by structuring complex information (Ainsworth, 2006).

Types of Representations

Different frameworks categorize representations in various ways. Bruner (1966) classifies them as enactive, iconic, and symbolic, reflecting developmental stages of learning. Lesh (1979) extends this classification to include real-life situations, concrete models, visual representations, symbolic expressions, and verbal descriptions. Later studies classify representations as graphical, tabular, algebraic, verbal, and pictorial (Herbel-Eisenmann, 2002; Cleaves, 2008).

These classifications highlight that no single representation is sufficient; rather, understanding emerges from the coordination of multiple forms.

Representational Fluency and Transitions

A central component of this framework is representational fluency, defined as the ability to interpret, translate, and connect different representations (Lesh et al., 2003).

Recent studies emphasize that difficulties in learning mathematics often stem not from understanding individual representations, but from challenges in transitioning between them (McCulloch et al., 2022). Students frequently experience difficulties in: maintaining consistency between variables, interpreting graphical structures, translating symbolic expressions into meaningful contexts (Adom, 2020; Didiş et al., 2016).

Role of Representations in Algebra Learning

Algebra is a domain where representations play a particularly critical role due to its abstract and symbolic nature (Kaput, 1989). Representations help students connect symbolic expressions with meaning and support algebraic reasoning (Blanton & Kaput, 2005).

Graphical and tabular representations are especially important for understanding functional relationships; however, students often struggle to interpret these representations meaningfully (Stylianou, 2010).

Errors in Representational Transitions

Errors in representational transitions are not random but systematic and indicative of underlying conceptual misunderstandings. Research identifies several common error types: focusing on visual appearance rather than mathematical structure misinterpreting slope and intercept inconsistent variable usage incorrect mapping between representations (McCulloch et al., 2022; Adom, 2020).

Technology and Representations

Digital tools provide dynamic representations that can support conceptual understanding. However, recent findings suggest that technology alone is insufficient; without proper instructional guidance, students may still fail to establish meaningful connections between representations (Gülkilik et al., 2023). This study uniquely contributes to the literature by systematically investigating representational transitions specifically at the 7th and 8th grade levels in Türkiye, highlighting both the types of errors and the cognitive challenges that emerge in middle school algebra.

Methodology

Research Design

A single-case explanatory case study design was used to examine 7th- and 8th-grade students' transitions between multiple representations and the errors they make during this process. The primary reason for choosing a case study as a research design was the need to gain an in-depth and contextual

understanding of the cognitive processes students experience when transitioning between representations and the nature of the errors that occur during this process. According to Creswell (2021), a case study is a qualitative research design that allows for a detailed examination of a situation or phenomenon within its context over a specific period of time (p. 99). In this context, it is an appropriate design for this study, which seeks to answer "how" and "why" questions such as "how do students make these transitions?" and "what are the types of errors?" because it offers a powerful research approach in situations where the boundaries between phenomenon and context are unclear and where participants' individual thought processes and experiences are central.

Here, the "case" is defined as the individual student's process of transitioning between algebraic, graphical, tabular, and verbal representations in solving mathematics problems. Unlike quantitative methods, case studies provide the opportunity to reveal the complexity of student thinking processes and describe the possible underlying causes of errors (conceptual, procedural, etc.) in detail through rich data such as interviews, observations, and document analysis.

Research Participants

The study was conducted at a middle school affiliated with the Ministry of National Education in Sinop province. The study group was formed using "convenience sampling," a purposive sampling method. Convenience sampling ensures the proximity of the study group to the researcher(s) conducting the study and ease of access (Patton, 2014). Fifty-four students participated in the study: 24 seventh-grade students and 30 eighth-grade students. The 8th-grade students selected for the study group were coded as S.8.1, S.8.2, ..., S.8.30, while the 7th-grade students were coded as S.7.1, S.7.2, ..., S.7.24. This also ensured that the students' identities remained confidential.

Data Collection

To examine students' transitions between algebraic representations and those involving graphs, tables, and verbal representations, the researchers used a data collection tool consisting of 10 main questions and 14 subquestions. The frequencies and percentages of the questions used, based on different representations, are presented in Table 1.

Table 1

Frequencies and Percentages of Questions Regarding Transitions in Representations

Types of Representation Transitions	Frequency (f)	Percentage (%)
Transition from Algebraic to Graphical Representation	2	14,28
Transition from Graphical to Algebraic Representation	3	21,42
Transition from Algebraic to Tabular Representation	2	14,28

Transition from Tabular to Algebraic Representation	3	21,42
Transition from Algebraic to Verbal Representation	2	14,28
Transition from Verbal to Algebraic Representation	2	14,28
Total	14	100

According to Table 1, students were asked five questions each about algebraic representation, graphical representation, and tabular representation, while students were asked four questions about algebraic representation and verbal representation. An example of these questions is presented in Figure 1.

Figure 1.

Example of a Problem Item

2. $\frac{x}{15} + \frac{x}{20} = 21$

The equation above shows how Cemil and Ahmet read the same book daily.

- Explain the number of pages Ahmet and Cemil read daily by showing them in a table.
- Explain the number of pages Ahmet and Cemil read graphically.
- Express the above equation verbally.

When preparing the questions, first, 7th- and 8th-grade textbooks, question books developed by the Ministry of National Education, and literature were reviewed. The questions were then prepared and evaluated through a pilot study with 5 students to ensure clarity and appropriateness of the items. Based on these opinions, the questions were revised and finalized. Data collection, conducted in a natural classroom setting under the supervision of two researchers and the relevant classroom teacher, consisted of two class hours with each study group.

In addition to written problem-solving tasks, semi-structured interviews were conducted individually with each student to gain deeper insights into their reasoning and transition strategies between representations. Interview questions focused on students' thought processes, difficulties encountered, and explanations for errors.

Analysis of Data

The qualitative data obtained from students' solution sheets and semi-structured interviews were analyzed through content analysis. Content analysis enables researchers to systematically categorize and interpret qualitative data by organizing them under meaningful themes and concepts (Yıldırım & Şimşek, 2021).

The analysis process followed a systematic and iterative procedure. First, all raw data were independently reviewed by three researchers to gain familiarity with the dataset. During this initial

phase, preliminary codes were generated to capture students' transition processes between representations (e.g., algebraic to graphical, graphical to algebraic, verbal to algebraic) and the types of errors encountered in these processes. Based on this preliminary coding, a comprehensive coding framework was collaboratively developed. The dataset was then re-examined and coded according to this framework. Throughout the analysis, constant comparison between 7th- and 8th-grade students' data was employed to identify similarities, differences, and trends in errors and transition strategies. The themes were finalized by grouping related codes under broader conceptual categories representing representation transitions and problem-solving errors.

Validity and Reliability

To enhance the trustworthiness of the study, strategies addressing credibility, dependability, confirmability, and transferability were implemented. Credibility was ensured through researcher triangulation. Three researchers independently examined and coded the data, minimizing individual bias and increasing the accuracy of interpretations. To establish dependability, inter-coder reliability was calculated. Thirty percent of the dataset was independently coded by the three researchers. The agreement rate was calculated using the formula suggested by Miles and Huberman (1994): $\text{Reliability} = \text{Consensus} / (\text{Consensus} + \text{Disagreement}) \times 100$

The inter-coder reliability coefficient was found to be 92%, indicating a high level of consistency among coders. Following independent coding, discrepancies were discussed in joint evaluation meetings. Through consensus-based revisions, the coding framework was refined and finalized. This collaborative review process also strengthened confirmability, as interpretations were grounded directly in the data rather than individual assumptions.

Finally, to support transferability, detailed descriptions of the research context, participants, data collection tools, and analysis procedures were provided, enabling readers to assess the applicability of the findings to similar contexts. These procedures collectively enhanced the methodological rigor and trustworthiness of the study.

Results

Within the scope of this study, the success and error tendencies of 7th and 8th grade students in transitions among algebraic, graphical, tabular, and verbal representations were examined. The findings reveal student performance, types of errors made, and the frequency of these errors for each type of representation transition.

Transition from Algebraic to Graphical Representations and the Transition from Graphical to Algebraic Representations

When transitioning from algebraic to graphical representations, 27 (25%) of all students answered two problems correctly, 56 (51.85%) gave blank answers, and 15 (13.88%) gave incorrect

answers. Of the 7th grade students' answers, 12 (25%) were correct, 28 (58.33%) gave blank answers, and 8 (16.6%) were incorrect. Of the 8th grade students' answers, 15 (25%) were correct, 38 (63.33%) gave blank answers, and 7 (11.66%) were incorrect.

The most common mistake students made when graphically representing algebraic expressions was failing to draw the graph appropriately. Seventh-grade students, in particular, drew graphs or produced fixed graphs without analyzing the algebraic expression. Both seventh- and eighth-grade students struggled with data placement and selecting the graph type.

Table 2
Types of Errors Made by 7th and 8th Grade Students in the Transition from Algebraic to Graphical Representation

Grade level	Question number	True	Blank	False	Total	Identified Error Types			Total
						Drawing graphs that do not correspond to values in the algebraic expression	Placing data incorrectly	Drawing the graph incorrectly	
7th Grade	Question 2	11	11	2	24	1	0	1	2
	Question 5	1	17	6	24	5	1	1	7
	Total	12	28	8	48	6	1	2	9
8th Grade	Question 2	7	21	2	30	2	0	0	2
	Question 5	8	17	5	30	2	2	2	6
	Total	15	38	7	60	4	2	2	8
Grand Total		27	56	15	108	10	3	4	17

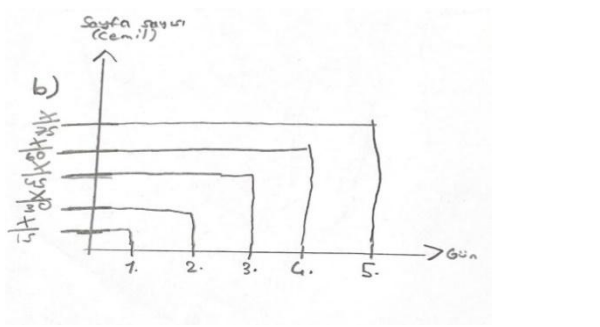
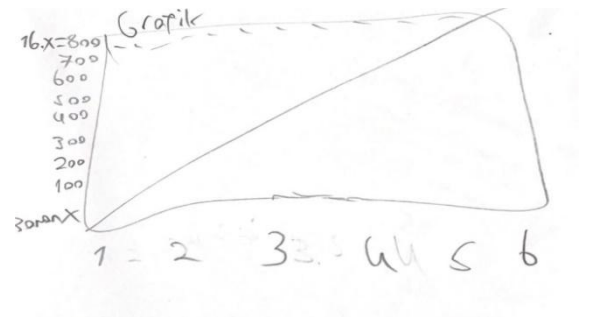
According to the data in Table 2, the types of errors students made when transitioning from algebraic to graphical representation were: drawing graphs that do not correspond to the values in the algebraic expression, placing data incorrectly, drawing the graph incorrectly, and placing data but not drawing the graph.

The most common error made by 7th grade students when transitioning from algebraic to graphical representation was drawing graphs that do not correspond to the values in the algebraic

expression (66.66%), while other common errors were drawing graphs incorrectly (22.22%) and placing data incorrectly (11.11%). There were no errors in placing data but not drawing the graph.

Figure 2.

Examples of the Types of Errors Made by 7th Grade Students in the Transition from Algebraic to Graphical Representation

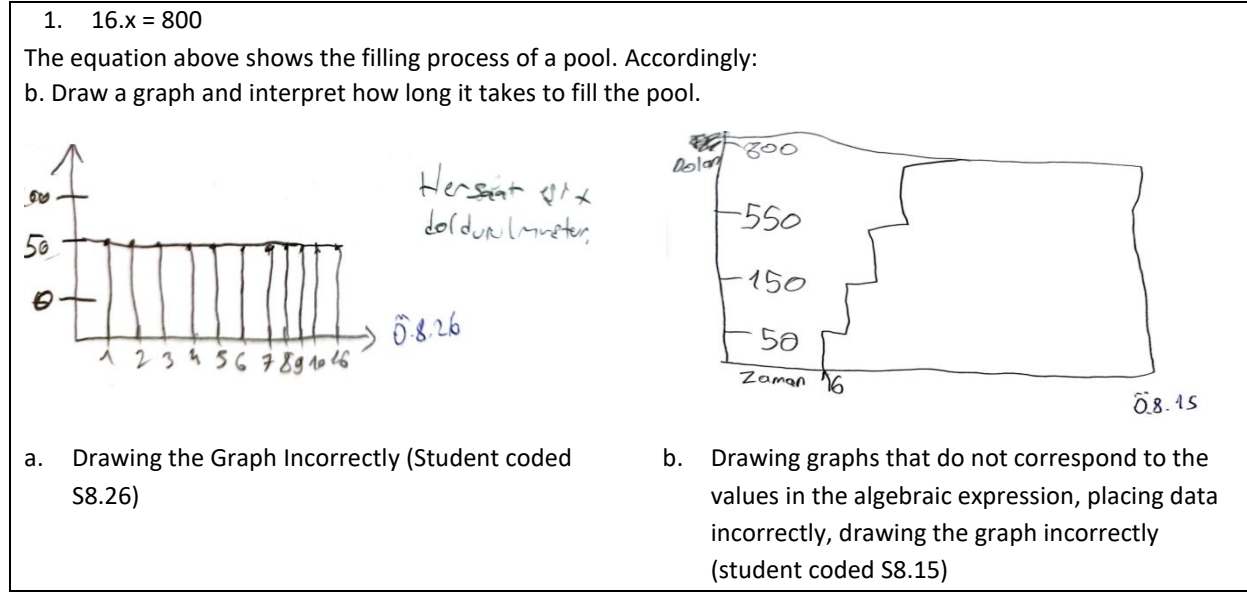
<p>2. $\frac{x}{15} + \frac{x}{20} = 21$</p> <p>The equation above shows how Cemil and Ahmet read the same book daily.</p> <p>b. Explain the number of pages Ahmet and Cemil read graphically.</p>	<p>5. $16.x = 800$</p> <p>The equation above shows the filling process of a pool. Accordingly:</p> <p>b. Draw a graph and interpret how long it takes to fill the pool.</p>
	
<p>a. Drawing a graph that does not correspond to the data in the algebraic expression (Student with code S.7.19)</p>	<p>b. Incorrect data placement, incorrect graph drawing (Student with code S.7.22)</p>

In Figure 2, the student coded S.7.19, without analyzing the algebraic expression given in Question 2, used $x/15$ as the number of pages Ahmet reads daily and $x/20$ as the number of pages Cemil reads daily, and plotted them on the graph accordingly. This also does not match the graph requested in the question. The student coded S.7.22 did not understand the purpose of the algebraic expression given in the question and wrote the portion of the pool filled in the column according to his own specified intervals. He also made an error by incorrectly specifying the algebraic expression and claiming that the pool would fill in 6 hours.

The most common error made by 8th-grade students when transitioning from algebraic representation to graphical representation was drawing a graph that did not correspond to the values in the algebraic expression (50.00%), while other common errors were placing the data incorrectly (25.00%) and drawing the graph incorrectly (25.00%). The least common error was placing the data but not drawing the graph (11.11%). In Figure 3, the student with code S.8.26 did not show the increase in the occupancy of the pool while drawing the graph, but drew a fixed graph.

Figure 3.

Examples of the Types of Errors Made by 8th Grade Students in the Transition From Algebraic to Graphical Representation



Student S.8.15, on the other hand, did not place the data in an orderly manner. He was unable to represent the pattern in the algebraic expression on the graph. According to student S.8.15's graph, the pool's occupancy rate at hour 16 was 50 m^3 . He made the error of drawing a graph that did not correspond to the values in the algebraic expression.

When 7th and 8th grade students were considered together, it was determined that while 7th grade students did not make the error of placing data but not drawing the graph, 8th grade students made this error. The most common error in both groups was drawing a graph that did not correspond to the values in the algebraic expression.

When the errors made by 7th and 8th grade students when transitioning from algebraic to graphical representations were generally considered, it was determined that, although there were minor differences, the error trends were similar. For example, the most common error in both groups when transitioning from tabular to graphical representation was placing data but not drawing the graph. Similarly, the most common error in transitioning from algebraic to graphical representation was drawing a graph that did not correspond to the values in the algebraic expression. However, there are exceptional cases, such as the error of "Drawing the Graph Incorrectly" being made significantly more frequently in 7th graders than in 8th graders when transitioning from verbal to graphical representation. When transitioning from graphical to algebraic representation, 10 (6.17%) of the students' answers to the three problems were correct, 119 (73.45%) were blank, and 33 (20.37%) were incorrect. Of the 7th grade students' answers, 4 (5.55%) were correct, 45 (62.5%) were blank, and 23

(31.95%) were incorrect. Of the 8th grade students' answers, 6 (6.66%) were correct, 74 (82.22%) were blank, and 10 (11.11%) were incorrect.

Table 3

Types of Errors Made by 7th and 8th Grade Students in the Transition From Graphical to Algebraic Representation

Grade level	Question number	True	Blank	False	Total	Identified Error Types					Total
						Creating an algebraic expression that does not correspond to the graphical representation	Solving numerically without creating an algebraic expression	Ignoring the units on the x-axis	Creating an incomplete algebraic expression	Using the wrong operation (such as subtraction instead of addition)	
7th Grade	Question 1	1	14	9	24	2	6	0	1	0	9
	Question 7	2	10	12	24	2	7	0	2	2	13
	Question 9	1	21	2	24	0	2	0	0	0	2
	Total	4	45	23	72	4	15	0	3	2	24
8th Grade	Question 1	2	27	1	30	0	0	0	1	0	1
	Question 7	2	23	5	30	3	2	0	0	1	6
	Question 9	2	24	4	30	0	1	3	0	0	4
	Total	6	74	10	90	3	3	3	1	1	11
Grand Total		10	119	33	162	7	18	3	4	3	35

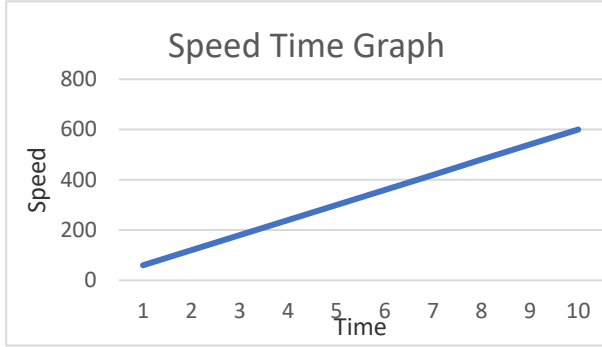
According to the data in Table 3, the errors students made when switching from graphical to algebraic representations were: creating an algebraic expression that does not correspond to a verbal expression, solving numerically without using an algebraic expression, ignoring the units on the x-axis, creating an incomplete algebraic expression, and using an incorrect operation.

The most common errors made by 7th-grade students when switching from graphical to algebraic representations were solving numerically without creating an algebraic expression (62.5%), creating an algebraic expression that does not correspond to a graphical representation (16.66%), creating an incomplete algebraic expression (12.5%), and using an incorrect operation (8.33%).

Figure 4.

Examples of the Types of Errors Made by 7th Grade Students When Switching from Graphical to Algebraic Representation

1. The graph below shows the speed-time graph of Onur's journey.

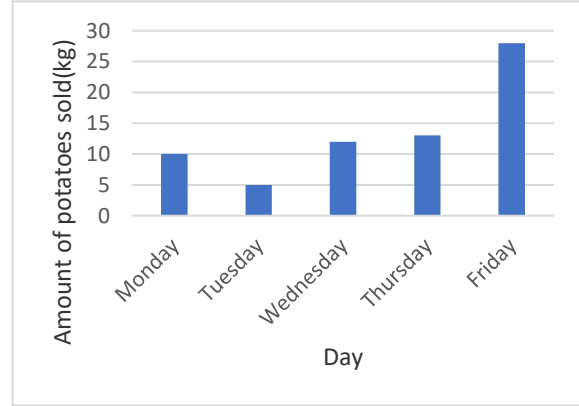


a. Create and solve the algebraic expression for the distance traveled with respect to time.

$$\frac{60}{1} = \frac{150}{2} = \frac{300}{3} = \frac{450}{7}$$

a. Solving numerically without creating an algebraic expression, using incorrect operations (S.7.12)

7. Ayhan, the greengrocer, buys potatoes for 25 TL per kilo and sells them for 30 TL. The chart below shows the daily amount of potatoes Ayhan sells each weekday.



a. Based on the information provided above, write and solve the algebraic expression representing the amount of profit Ayhan the greengrocer made.

$$\begin{aligned} a &= x = 9 + 5 + 12 + 14 + 28 \\ x &= 26 + 39 \\ x &= 65 \text{ kg} \end{aligned}$$

b. Solve numerically without using algebraic expressions (S.7.8)

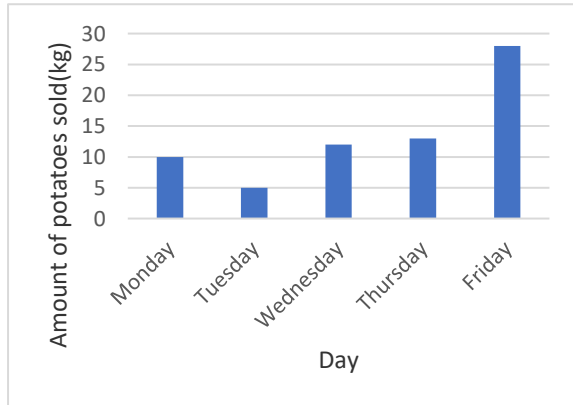
According to Figure 4, student S.7.12 used numerical expressions that did not correspond to the graphical representation. Student S.7.8 solved the question using only numerical expressions, even though the question asked for a transition from graphical to algebraic representation.

The most common errors made by 8th-grade students when transitioning from graphical to algebraic representation were creating an algebraic expression that did not correspond to a verbal expression (27.27%), solving numerically without creating an algebraic expression (27.27%), and ignoring the units on the x-axis (27.27%). Other errors included creating an incomplete algebraic expression (9.09%) and using incorrect operations (9.09%). Figure 6 provides examples of the types of errors made by 8th-grade students when transitioning from graphical to algebraic representation.

Figure 5.

Examples of the Types of Errors Made by 8th Grade Students When Switching from Graphical to Algebraic Representation

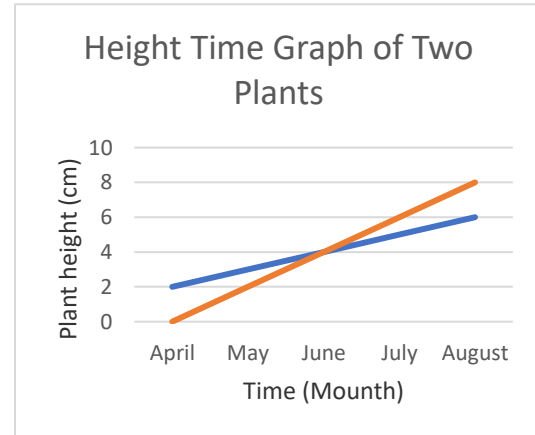
7. Ayhan, the greengrocer, buys potatoes for 25 TL per kilo and sells them for 30 TL. The chart below shows the daily amount of potatoes Ayhan sells each weekday.



- a. Based on the information provided above, write and solve the algebraic expression representing the amount of profit Ayhan the greengrocer made.

- a. Creating an algebraic expression that does not correspond to a graphical representation (student coded S.8.27)

9. The graph below shows the status of two plants over time.



Based on the information provided above:

- a. Express algebraically the time-dependent growth of two plants. Find the location where their heights are equal by solving the equations you obtained.

- b. Not taking into account the units on the X-axis (student coded S.8.16)

According to Figure 5, student S.8.27 attempted to create an algebraic expression that did not correspond to the graphical representation. Student S.8.16, on the other hand, created an algebraic expression expressing the amount of growth by only taking the heights of the plants in the first and last months into account. He disregarded other months and factors such as the slope of the graph. He ignored other data on the x-axis.

When 7th and 8th grade students are considered together, the error of solving numerically without creating an algebraic expression occurred much more frequently in 7th grade students than in 8th grade students, while the error of ignoring the units on the x-axis was a type of error that

occurred only in 8th grade students. Creating an algebraic expression that did not correspond to the graphical representation, creating an incomplete algebraic expression, and using an incorrect operation were identified as errors that occurred at similar frequencies in both groups. This finding supports previous research by Goldin & Shteingold (2001) and Arcavi (2003), indicating that students struggle to understand context and connect symbols with context when transitioning between mathematical representations.

Transition from Algebraic to Tableau and from Tableau to Algebraic Representation

When transitioning from algebraic to tableau, 24 (22.22%) of the student answers to the two problems posed were correct, 72 (66.66%) were blank, and 12 (11.11%) were incorrect. Of the 7th-grade student answers, 13 (27.08%) were correct, 25 (52.08%) were blank, and 10 (20.83%) were incorrect. Of the 8th-grade student answers, 11 (18.33%) were correct, 47 (78.33%) were blank, and 2 (3.33%) were incorrect.

Table 4

Types of Errors Made by 7th and 8th Grade Students When Transitioning from Algebraic to Table

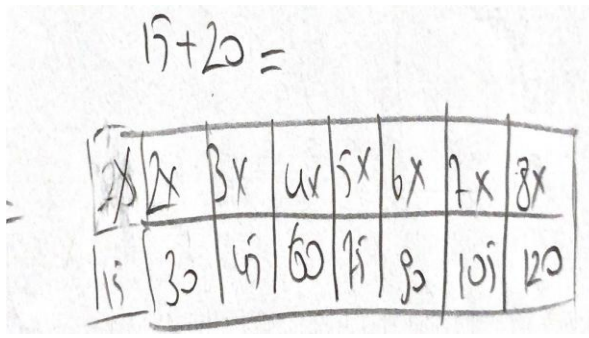
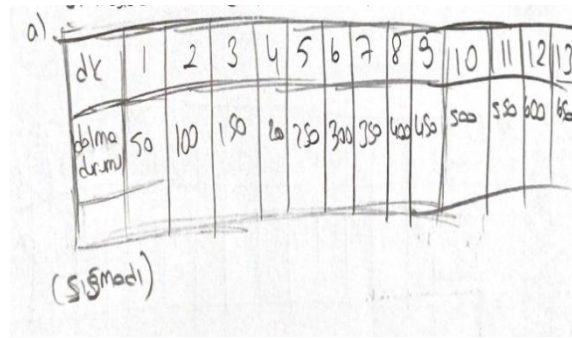
Grade level	Question number	True	Blank	False	Total	Identified Error Types		Total
						Incorrectly establishing data relationships	Not naming Rows and Columns (Drawing error)	
7th Grade	Question 2	12	7	5	24	4	4	8
	Question 5	1	18	5	24	3	4	7
	Total	13	25	10	48	7	8	15
8th Grade	Question 2	6	24	0	30	0	0	0
	Question 5	5	24	1	30	1	1	2
	Total	11	47	2	60	1	1	2
Grand Total		24	72	12	108	8	9	17

According to the data in Table 4, the types of errors students made when transitioning from algebraic to tabular representation were identified as incorrectly establishing data relationships and not naming rows and columns.

7th grade students made the most errors of not naming rows and columns (53.3%). They then made the most errors of incorrectly establishing data relationships (46.66%). At the 8th grade level, they made one error each of incorrectly establishing data relationships and not naming rows and columns.

Figure 6.

Examples of Mistakes Made by 7th Grade Students in Transitioning to Algebraic Representation Table Representation Type

<p>$\frac{x}{15} + \frac{x}{20} = 21$</p> <p>2. The same book of Cemil and Ahmet shows the same reading situations on daily basis.</p> <p>a. Explain the number of pages that Ahmet and Cemil read daily by showing them.</p> 	<p>5. $16 \cdot x = 800$</p> <p>The equation above shows the filling status of a pool. Accordingly;</p> <p>a. Show and interpret the filling status of the pool by minute in a table.</p> 
<p>a. Incorrectly establishing the relationship between data, not naming rows and columns (student coded S.7.5)</p>	<p>b. Incorrectly establishing the data relationship (student coded S.7.3)</p>

In Figure 6, student S.7.5 failed to establish a relationship between the algebraic expression given in the question and the table. Student S.7.3 incorrectly established a relationship between the algebraic expression given in the question and the table, and created an incorrect table by confusing the pool's filling status with the time.

When considering 7th and 8th grade students together, it was determined that 7th grade students made significantly more errors than 8th grade students in the transition from algebraic to tabular representation. The errors identified in this section were "Incorrectly Establishing Data Relationships" (f=7) and "Not Naming Rows and Columns" (f=8), which were the most common errors made by 7th grade students. The errors "Incorrectly Establishing Data Relationships" (f=1) and "Not Naming Rows and Columns" (f=1) were only made once in question 5 by 8th grade students.

The most common error made by students when creating a table from algebraic expressions was skipping the tabulation stage by performing a numerical solution. Error types such as incorrect operations and equation selection were observed in 8th graders, while 7th graders mostly preferred direct numerical solutions. Difficulties were noted, particularly with tabular representation of two-variable expressions.

Of the three questions regarding the transition from tabular to algebraic representation, 20

(12.34%) of the student answers were correct, 103 (63.58%) were blank, and 37 (22.83%) were incorrect. Of the 7th grade student answers, 9 (12.5%) were correct, 45 (62.5%) were blank, and 18 (25%) were incorrect. Of the 8th grade student answers, 11 (12.22%) were correct, 58 (64.44%) were blank, and 19 (21.11%) were incorrect.

Table 5

Types of mistakes made by 7th and 8th grade students in the transition from table representation type to algebra representation type

Grade level	Question number	True	Blank	False	Total	Identified Error Types				Total
						Creating an algebraic expression that does not correspond to a table representation	Solving numerically without creating an algebraic expression	Using the wrong operation (such as using subtraction instead of addition)	Using two-variable equations	
7th Grade	Question 4	4	12	8	24	2	6	0	0	8
	Question 8	3	15	6	24	2	3	1	0	6
	Question 10	2	18	4	24	4	0	0	0	4
	Total	9	45	18	72	8	9	1	0	18
8th Grade	Question 4	4	19	7	30	3	4	2	0	9
	Question 8	5	20	5	30	5	0	1	0	6
	Question 10	3	20	7	30	6	0	1	1	8
	Total	11	58	19	90	14	4	4	1	23
Grand Total	20	103	37	162	22	13	5	1	41	

According to the data in Table 5, the errors students made when switching from tabular to algebraic representations were: creating an algebraic expression that does not correspond to the tabular representation, solving numerically without creating an algebraic expression, using incorrect operations, and using two-variable equations.

The most common error made by 7th grade students when switching from tabular to algebraic representations was solving numerically without creating an algebraic expression (50%), while other errors included creating an algebraic expression that does not correspond to the tabular representation (44.44%) and using incorrect operations (5.55%).

Figure 7.

Examples of the Types of Mistakes 7th Grade Students Make When Transitioning from Tabular Representation to Algebra Representation Type

Day	1	2	3	4	5	6	7
Planted Field (m^2)	20	5	5	5	5	5	5

The field above shows Nisa's daily planting status. Accordingly;

a. Write and solve the algebraic expression that shows how many meters of area Nisa planted.

$$a = 6 \times 5 + 20$$

a. Solving numerically without creating an algebra representation (Student coded S.7.4)

8. The daily walking times of Firdevs and İlknur are given in the table below. According to the information given above;

a. Show algebraically the daily walking situations of Firdevs and İlknur.

Days	1. day	2. day	3. day	4. day
Firdevs	30	50	70	90
İlknur	25	55	85	115

a)

$$F = (30x) + (50x) + (70x) + (90x) \quad (25x) + (55x) + (85x) + (115x)$$

b)

Firdevs İlknur

b. Using the wrong operation (Student with code S.7.16)

10. The table below shows the absenteeism status of students in the secondary school where Ali Principal was newly appointed in the last 4 years.

a. Express and interpret the students' absenteeism in the last 4 years algebraically.

Year	5. year	6. year	7. year	8. year
absentee statuses	25	30	35	40

$$40 + 30 + 35 + 25 = 130$$

c. Solving numerically without creating an algebra representation (student coded S.7.9)

In Figure 7, the student coded S.7.4 and the student coded S.7.9 solved the question without creating an algebraic expression. The student coded S.7.16 used the wrong operation while creating the algebraic expression.

While the most common mistake made by 8th grade students when switching from tabular representation type to algebraic representation type was determined as creating an algebraic expression that does not correspond to the table representation (60.86%), the other mistakes they made were solving numerically without creating an algebraic expression (17.37%), using the wrong operation (17.37%) and using two-variable equations (4.34%). Figure 5 shows examples of the types of mistakes 8th grade students made when transitioning from table representation type to algebra

representation type.

Figure 8.

Examples of the Types of Mistakes 8th Grade Students Make When Switching From Tabular Representation Type to Algebra Representation Type

Day	1	2	3	4	5	6	7
Cultivated Field (m^2)	20	5	5	5	5	5	5

The field above shows Nisa's daily planting status. Accordingly;

a. Write and solve the algebraic expression that shows how many square meters of land Nisa planted.

Handwritten solution for student Ö.8.18:

$20,6.5 = 20,30 = 600m^2$ olarak tarla ekmiştir
Ö.8.18

Handwritten solution for student Ö.8.22:

$9 \rightarrow (x+1) = 5 \Rightarrow (x+1) = 5$
 $x = 5-1$
 $x = 4$ Ö.8.22

a. Solving numerically without creating an algebraic expression, using the wrong operation (Student coded S.8.18)

b. Creating an algebraic expression that does not correspond to a table representation (student with code S.8.22)

According to Figure 8, the student coded Ö.8.18 tried to solve the question containing a table without using algebraic expressions and tried to add the field planted in the next 6 days to the field planted on the first day. However, in his solution, he made the mistake of using the wrong operation by doing the operation $20.6.5$ instead of $20+(6.5)$. The student coded Ö.8.22 created an algebraic expression that did not correspond to the data in the table.

When 7th and 8th grade students are taken together; It was determined that 8th grade students made more mistakes in creating algebraic expressions that did not correspond to the table representation than 7th grade students. The error of using two-variable equations was determined to be a mistake that only occurred in 8th grade students. The error of solving numerically without creating an algebraic expression was determined to be a mistake that occurs more frequently in 7th graders than in 8th grades. The error of using the wrong operation was a type of error that occurred more frequently in the 8th grade. These findings indicate that students struggle to convert multivariable data relationships from tables into algebraic expressions and tend to prefer direct numerical solutions over forming algebraic expressions. This suggests that concrete-operational representation processes, as proposed by Dienes (1960) and Confrey (1990), are not adequately supported.

Transition From Algebra Representation To Verbal Representation And Transition From Verbal Representation To Algebra Representation

In the transition from algebra representation to verbal representation, 11 (10.18%) of the answers given by the students to the 2 problems were correct, 68 (62.96%) were blank, and 29 (26.85%) were incorrect. Of the answers given by 7th grade students, 9 (18.75%) are correct, 29 (60.41%) are blank and 10 (20.83%) are wrong. Of the answers given by 8th grade students, 2 (3.33%) were determined as blank, 39 (65%) and 19 (31.66%) were incorrect.

When students were asked to explain algebraic expressions verbally, it was observed that they could not understand the context of the given algebraic expression and made incorrect or incomplete explanations. 8th grade students made more mistakes in this transition, especially the lack of support with numerical data and uncertainty in verbal expression were noted.

Table 6
Types of Mistakes Made by 7th and 8th Grade Students in the Transition from Algebraic Representation Type to verbal representation type

Grade level	Question number	True	Blank	False	Total	Identified Error Types							Total
						Incomplete verbal expression of the data in the algebraic expression	Expressing any numerical data (verbal)	Not using any numerical data (verbal)	Adding data that is not in the algebraic expression when writing verbally	Expressing the solution of the equation verbally	Expressing the equation mathematically	Expressing the equation semi-mathematically	
7th Grade	Question 2	6	12	6	24	1	1	0	2	2	0	0	6
	Question 5	3	17	4	24	0	0	0	3	1	0	0	4
	Total	9	29	10	48	1	1	0	5	3	0	0	10
8th Grade	Question 2	1	21	8	30	2	2	1	1	3	2	2	13
	Question 5	1	18	11	30	0	0	0	11	10	0	0	21
	Total	2	39	19	60	2	2	1	12	13	2	2	34
Grand Total		11	68	29	108	3	3	1	17	16	2	2	44

According to the data in Table 6, the types of errors students made when transitioning from algebraic to verbal representation were identified as: verbally expressing incomplete data in the

algebraic expression, semi-algebraically expressing it, using no numerical data (verbally), adding data not present in the algebraic expression while writing verbally, verbally expressing the solution of the equation, expressing the equation mathematically, and expressing the equation semi-mathematically.

The most common error made by 7th grade students when transitioning from algebraic to verbal representation was adding data not present in the algebraic expression while writing verbally (50%). Other most common errors were verbally expressing the solution of the equation (30%), semi-algebraically expressing it (10%), and verbally expressing incomplete data in the algebraic expression (10%). They did not make the errors of not using any numerical data, expressing the equation mathematically, or expressing the equation semi-mathematically.

Figure 9.

Examples of the Types of Errors Made by 7th Grade Students in the Transition from Algebraic to Verbal Representation

<p>$16.x = 800$</p> <p>The equation above shows the filling status of a pool. Accordingly:</p> <p>c. Verbally describe the filling status of the pool.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>The volume of the pool is 800 dm³. The pool fills in 50 minutes, on average 1 hour.</p> </div> <p>Adding data not present in the algebraic expression when writing verbally (Student with code S.7.11)</p>
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In Figure 9, the student coded S.7.11 made an error by confusing the concepts of time and volume when converting the given algebraic expression to a verbal expression.

The most common errors made by 8th-grade students in the transition from algebraic to verbal representation were: verbally expressing the solution to the equation (38.23%) and adding data not present in the algebraic expression while writing verbally (35.29%). Other errors included: verbally expressing incomplete data in the algebraic expression (5.88%), semi-algebraically expressing the equation (5.88%), mathematically expressing the equation (5.88%), semi-mathematically expressing the equation (5.88%), and not using any numerical data (verbally) (2.94%). Figure 10 shows examples of the types of errors made by 8th-grade students in the transition from algebraic to verbal representation.

Figure 10.

Examples of the Types of Errors Made by 8th Grade Students in the Transition from Algebraic to Verbal Representation

The figure shows two examples of student work. Example a (left) shows a student's handwritten response to a problem. The student has written 'Barbaros'un', '100 kedi', 'olarak göster', 'Cemil'e', 'arasında (3) farktır.', and 'S.8.16'. There are some scribbles and a '6' with a horizontal line above it. Example b (right) shows a student's handwritten response to a problem. The student has written 'Cemil'in günlük kitap okuma durumu bir sayının 15'e bilini', 'Ahmet'in günlük kitap okuma durumu bir sayının 20'ye bilini', 'bu ikisinin toplamı 21 miş', and 'S.8.27'. Below the text is the label 'b. Expressing the equation semi-mathematically (student coded Ç.8.27)'.

In Figure 10, the student coded S.8.16 added data, such as a multiple of 3, when converting the algebraic expression given in the question into a verbal expression. However, he was unable to translate the data in the algebraic expression into a verbal expression and left it incomplete. The student coded S.8.27, on the other hand, attempted to verbally express the algebraic expression given in the question using expressions such as "Cemil's daily book reading," but also used mathematical expressions such as "dividing a number by 15." Indeed, he attempted to verbally transform the algebraic expression into a semi-mathematical expression.

When considering 7th and 8th grade students together, errors such as adding data not included in the algebraic expression while writing verbally and verbally expressing the solution to the equation were made much more frequently by 8th grade students than by 7th grade students. The errors made by both groups were: incomplete verbal expression of data in algebraic expression, semi-algebraic expression, no numerical data (verbal), expressing the equation mathematically, and expressing the equation semi-mathematically, and were made at similar frequencies.

Of the two problems related to the transition from verbal to algebraic representation, 16 (14.81%) of the student answers were correct, 61 (56.48%) were blank, and 31 (28.70%) were incorrect. Of the answers given by 7th grade students, 4 (8.33%) were correct, 27 (56.25%) were blank, and 17 (35.41%) were incorrect. Of the answers given by 8th grade students, 12 (20%) were correct, 34 (56.66%) were blank, and 14 (23.33%) were incorrect.

Table 7

Types of Errors Made by 7th and 8th Grade Students When Transitioning From Verbal to Algebraic Representation

Grade level	Question number	True	Blank	False	Total	Identified Error Types			Total
						Creating an algebraic expression that does not correspond to a verbal expression	Solving numerically without creating an algebraic expression	Making operational errors (unit conversion)	
7th Grade	Question 3	2	12	10	24	3	4	3	10
	Question 6	2	15	7	24	0	7	0	7
	Total	4	27	17	48	3	11	3	17
8th Grade	Question 3	3	17	10	30	2	6	2	10
	Question 6	9	17	4	30	0	4	0	4
	Total	12	34	14	60	2	10	2	14
Grand Total		16	61	31	108	5	21	5	31

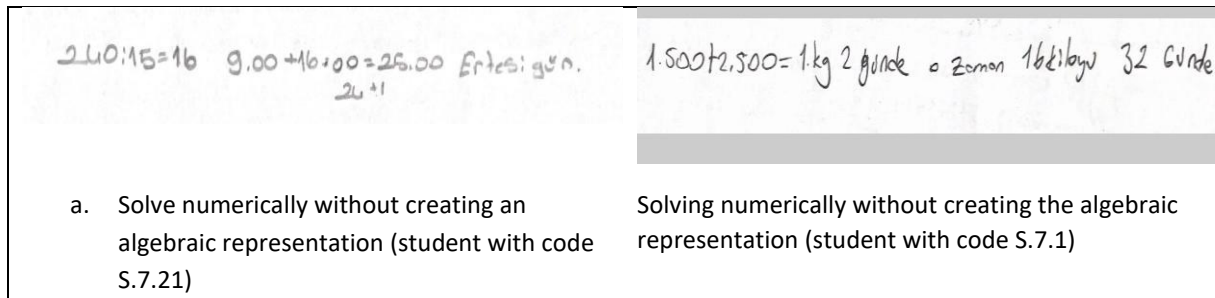
According to the data in Table 7, the errors students made when switching from verbal to algebraic representation were identified as creating an algebraic expression that does not correspond to the verbal expression, solving numerically without creating the algebraic expression, and making computational errors (unit conversion).

The most common error made by 7th grade students when switching from verbal to algebraic representation was solving numerically without creating the algebraic expression (64.70%). Other errors included creating an algebraic expression that does not correspond to the verbal expression (17.65%) and making computational errors (unit conversion) (17.65%).

Figure 11.

Examples of the Types of Errors Made by 7th Grade Students in the Transition from Verbal to Algebraic Representation

<p>6. Coffee maker Veli sells an average of 15 cups of tea per hour from his samovar, which holds 240 cups of tea. Mr. Veli brews tea at 9:00 a. At what time does he brew his next cup of tea?</p> <p>a. Write and solve the algebraic expression showing how long it will take for the tea in Mr. Veli's samovar to finish.</p>	<p>3. Ms. Ayşe went to a dietitian to lose weight. Ms. Ayşe's initial weight was 96 kg. Ms. Ayşe and her dietitian prepared a diet plan that would allow her to lose 500 g of weight per day. Given that Ms. Ayşe's desired weight is 80 kg, find the following:</p> <p>a. Express algebraically how many days it will take Ms. Ayşe to reach her desired weight, and solve.</p>
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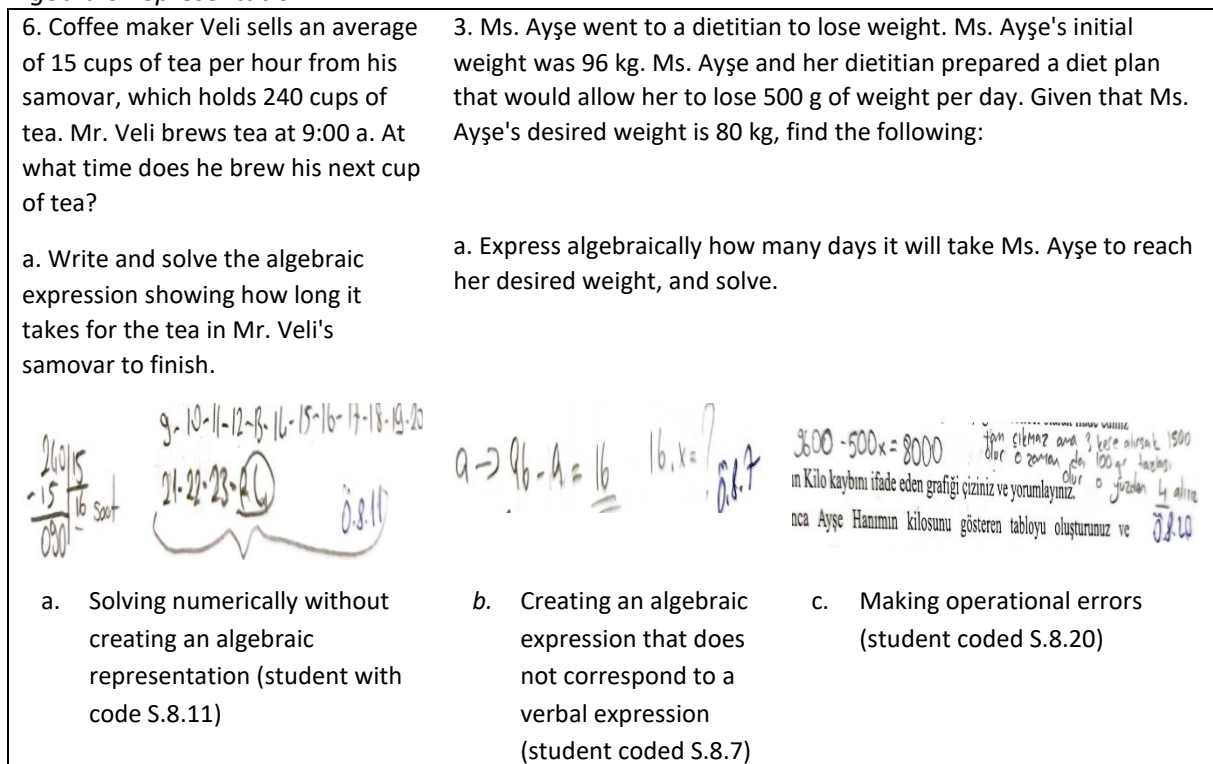


According to Figure 11, student S.7.21 did not use algebraic expressions but resorted to a direct numerical solution. This error was made because we asked them to represent the problem as an algebraic expression. Student S.7.21, on the other hand, used verbal expressions instead of algebraic expressions when solving the problem.

The most common error made by 8th-grade students when transitioning from verbal to algebraic representation was solving numerically without creating an algebraic expression (71.42%). Other errors included creating an algebraic expression that did not correspond to the verbal expression (14.28%) and making an operational error (unit conversion) (14.28%). Figure 6 provides examples of the types of errors made by 8th-grade students when transitioning from verbal to algebraic representation.

Figure 12.

Examples of the Types of Errors Made by 8th Grade Students When Transitioning from Verbal to Algebraic Representation



According to Figure 12, student S.8.11 attempted to solve the verbal problem numerically and did not use any algebraic expressions. Student S.8.17, on the other hand, created an algebraic expression that was inappropriate for the given word problem. Student S.8.20 made a computational error (unit conversion) by writing 96 kilograms as 9600 grams and 80 kilograms as 8000 grams.

When 7th and 8th grade students are considered together, the most common error in both groups was solving numerically without creating an algebraic expression, which is a significant difference from other error types. Creating an algebraic expression that does not correspond to a verbal expression and making numerical errors were other error types observed at similar frequencies in both groups. This indicates that students have difficulties establishing verbal-mathematical relationships and struggle to connect symbolic and verbal representations during mathematical meaning-making processes (Blum & Leiss, 2007).

General Evaluation and High Rate of Blank Responses

Overall, the high rate of blank responses (average 50–65%) indicates that students face difficulties in representation transitions, attempt to proceed without understanding the topic, or fail to develop problem-solving strategies. Error types generally involve misalignments across representations, such as creating graphs or tables inconsistent with algebraic expressions, performing numerical solutions without forming algebraic expressions, or adding data to verbal statements.

These findings suggest that representation transitions are cognitively challenging for students and that instructional strategies should be structured to support these transitions with clear and concrete examples.

Relation to theoretical framework: The findings support the notion, within the framework of mathematical representation theory, that students exhibit weaknesses in processes of transforming, interpreting, and contextualizing representations. In particular, the findings confirm that the ability to transition between representation types, emphasized by Hiebert and Carpenter (1992) and Goldin (1998), is central to mathematical understanding.

Discussion and Conclusions

The findings of this study demonstrate that 7th- and 8th-grade students experience significant structural and conceptual difficulties when transitioning among algebraic, graphical, verbal, and tabular representations. Especially, the high error rates in transitions involving abstract and symbolic operations reveal insufficient representational fluency among students (Ainsworth, 1999; Duval, 2006). This indicates that representational transitions should not be treated merely as procedural conversions, but rather as processes of meaning construction grounded in conceptual understanding.

Notably, the high rate of unanswered items in transitions from graphical to algebraic forms indicates conceptual gaps in interpreting visual data and translating it into symbolic language. In line with Duval's (2006) view of representation as meaning-making, students appear to struggle with constructing mathematical meaning from visual information. Similar difficulties related to symbolization and mathematical language have been reported by Stylianou (2010) and Janvier (1987). These findings suggest that students face conceptual deficiencies in the process of mathematical meaning-making and require instructional support in representational transitions.

Students performed relatively better when converting algebraic expressions into graphical representations, possibly due to greater instructional exposure to graphing tasks (Leinhardt, Zaslavsky, & Stein, 1990). However, Yağız and Broutin (2023) reported that generating graphs posed significant challenges for 7th graders. This discrepancy may arise from differences in prior experience, task structure, or the integration of dynamic tools. Thus, instructional experience and opportunities for visualization emerge as critical factors influencing students' representational transition performance.

Regarding tabular representations, students were more successful in generating tables from algebraic expressions than in producing algebraic expressions from tables. While reading and organizing data seem manageable, generalizing patterns and expressing them symbolically require higher levels of abstraction and cognitive flexibility (Kaput, 1989; Confrey & Smith, 1994). The findings also show that students often provide numerical answers instead of constructing algebraic expressions, reflecting reliance on arithmetic reasoning rather than algebraic generalization (Booth, 1988; Kieran, 2007; Altun & Arslan, 2006; Umay, 2003). Additionally, mismatched graphs and inconsistent algebraic expressions indicate incomplete understanding of the structural constraints inherent in different representational systems (Duval, 2006).

Grade-level comparisons revealed that 7th graders provided more correct responses, whereas 8th graders left more items unanswered. This pattern may reflect affective factors such as increased academic pressure, anxiety, or reduced self-efficacy among older students (Pekrun et al., 2002). Sulhan, Bozkurt, and Isırgan (2024) similarly observed that older students tend to avoid responding when uncertain, indicating that emotional and motivational variables may significantly influence students' engagement with representational tasks.

Overall, representational fluency is not a procedural skill but a foundational component of mathematical learning intertwined with conceptual understanding and cognitive flexibility (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). The use of dynamic mathematics software (GeoGebra, Desmos) allows students to explore relationships among representations and strengthen symbolic–visual connections (Chen & Herbst, 2021; Hitt, González-Martín, & Mazón, 2023; Mosese &

Ogbonnaya, 2021). Instructional design should intentionally include structured activities that require students to work across multiple representations. Teacher professional development should emphasize strategies for guiding representational transitions and diagnosing related errors. Assessment practices should evolve to evaluate not only final answers but also the processes, strategies, and misconceptions evident in students' representational work. Scaffolded instruction, metacognitive activities, and classroom discussions are crucial for fostering students' critical and flexible mathematical thinking.

In conclusion, this study reinforces that the difficulties students face in representational transitions are conceptually grounded and that supporting students in constructing meaning across multiple representations can enhance algebraic thinking and deeper mathematical understanding. Future studies involving diverse age groups, curricula, and cultural contexts are necessary to better understand the developmental trajectory of representational fluency and the long-term effects of instructional interventions. Moreover, affective variables such as mathematics anxiety, self-efficacy, and perfectionism require closer examination, as they may significantly influence students' engagement with complex representational tasks and overall learning trajectories.

Limitations and Implications

While this study provides important insights into students' transitions among algebraic, graphical, verbal, and tabular representations, several limitations should be considered when interpreting the findings. The study was conducted with a specific group of 7th- and 8th-grade students within a particular educational context; therefore, the generalizability of the results may be limited. Students' experiences with representations are shaped by curricular structures, instructional approaches, and socio-cultural factors, suggesting that broader and more diverse samples are needed to strengthen external validity.

In addition, the study relied primarily on written responses to examine students' errors and representational transitions. Although written data provide valuable evidence of observable performance, they may not fully capture the underlying cognitive processes, reasoning strategies, and misconceptions that shape students' responses. Students' difficulties in transitioning between representations may involve more complex mental processes than those reflected in written work alone. Furthermore, the cross-sectional design of the study limits conclusions about the developmental progression of representational fluency. Observed differences between grade levels may reflect cohort-specific characteristics rather than true developmental change. Longitudinal studies are therefore needed to better understand how these competencies evolve over time.

Another important limitation concerns the role of affective factors. Although the findings

suggest that variables such as anxiety, avoidance, and self-efficacy may influence students' performance, particularly in the case of unanswered items, these factors were not directly measured. As a result, interpretations regarding students' emotional and motivational states remain tentative

Despite these limitations, the findings offer several important implications for both instructional practice and future research. First, the results indicate that representational fluency should be considered a central component of mathematics learning rather than a secondary or procedural skill. Instruction should be intentionally designed to include tasks that require students to translate, compare, and coordinate multiple representations. Particular emphasis should be placed on reverse transitions (e.g., graph-to-algebra, table-to-algebra), which were found to be especially challenging.

Moreover, instructional practices should move beyond procedural approaches and focus on supporting meaning-making processes. Teachers can facilitate this by explicitly addressing the relationships between representations, encouraging students to justify their reasoning, and incorporating activities such as error analysis and structured comparison tasks. Such approaches may help reduce common misconceptions and promote deeper conceptual understanding.

The integration of dynamic digital tools, such as GeoGebra or Desmos, also holds potential for strengthening connections between symbolic and visual representations. However, these tools should be embedded within carefully designed pedagogical frameworks to ensure that students engage in meaningful interpretation rather than superficial manipulation.

Teacher professional development is another critical area. Programs should focus on enhancing teachers' ability to identify students' representational difficulties and to design instruction that effectively addresses these challenges. Supporting teachers in understanding the cognitive demands of representational transitions is essential for improving classroom practice.

From a research perspective, future studies should adopt more comprehensive methodological approaches, such as mixed-method designs that include interviews, think-aloud protocols, and classroom observations, in order to better capture students' reasoning processes. Longitudinal research is particularly important for examining the development of representational fluency over time and the long-term effects of instructional interventions.

Finally, further research should explore the role of affective variables—such as mathematics anxiety, self-efficacy, and motivation—in shaping students' engagement with representational tasks. Comparative and cross-cultural studies may also provide valuable insights into how different educational contexts influence students' representational competencies and help inform the

development of more effective instructional approaches.

Research and Publication Ethics

In this study, all rules specified in the "Directive on Scientific Research and Publication Ethics of Higher Education Institutions" were followed. None of the actions specified under the second section of the Directive, "Actions Contrary to Scientific Research and Publication Ethics", have been carried out.)

Ethics committee permission information

Name of the committee that conducted the ethical assessment: Sinop University Human Research Ethics Committee

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1. Contribution rate statement of researchers: First Author 50% Second Author 25% Third Author %25
2. No potential conflict of interest was reported by the authors.

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